Difference between discounted payback and time to breakeven

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Abstract: Discounted payback is taught as a corollary to net present value in the capital budgeting chapter. Time-to-breakeven is taught in the time-value-money chapter which normally precedes the capital budgeting chapter. Due to myriad reasons, there are growing calls to simplify the curriculum by dropping the capital budgeting chapter as a requirement for non-finance majors in order to spend more time perfecting the delivery of the time-value-money chapter which by itself is a significant challenge to many students. This manuscript illustrates such calls are misplaced because the capital budgeting chapter is an essential chapter whose curricular contents include the discounted payback which is different and distinct from the time-to-breakeven taught in the time-value-money chapter.

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<div style="text-align: center;">**Difference between discounted payback and time to breakeven**</div>

Introduction

   Osborne (2010, p. 234) reported that the payback criterion dominated the NPV and IRR techniques in all European countries even though academics in finance had debated the relative superiority of NPV and IRR *ad nauseam*. We assume here that Osborne meant *discounted* payback and not *undiscounted* payback which is heretical to the basic tenet of time-value-money orthodoxy.

   This brief pedagogical note illustrates the importance of requiring the teaching of the capital budgeting chapter to all business students regardless of their chosen academic discipline(s). The mere teaching of time-value-money, TVM, concept is insufficient no matter how comprehensive its coverage. The note uses the discounted payback, DPB, learned in the capital budgeting chapter and compares it with the time-to-breakeven, TTB, learn in the TVM chapter. The note refutes recent attempts in curricular restructuring to jettison the capital budgeting chapter for non-finance majors, and it also refutes claims that a more-thorough and more-comprehensive coverage of the TVM concept would suffice to compensate the deletion of the capital-budgeting chapter for non-finance majors.

Illustrative example: Let's begin with a typical capital budgeting project with the given cash flows as: $CF_0 = -100$, $CF_1 = CF_2 = CF_3 = 40$, $CF_4 = 50$, and interest per year = 10%.

The CF key in the Texas Instruments® BA-II Plus Professional financial calculator will give us the discounted payback, DPB, as 3.0154 years. The same number can be easily proven in Microsoft® Excel spreadsheet as:

Year, t	CF_t	$PV(CF_t)$ at 10%	Cumulative $PV(CF_t)$
0	-100	-100	-100
1	40	36.36	-63.64
2	40	33.06	-30.58
3	40	30.05	-0.53
4	50	34.15	33.62

The DPB is obtained by interpolating as DPB = 3 + |-0.53|/34.15 = 3.01540. In interpolating, we assume that the $50 is spread evenly throughout the fourth year and does not occur discretely at the end of the fourth year. If the last cash flow occurs discretely at the end of the fourth year, then the correct answer for DPB is simply 4.000 years with an abandonment or sale value of $33.62 which coincidentally is also the project's net present value, NPV.

What if the given cash flows are viewed simply as the time-value-money problem by someone else who has not learned capital-budgeting chapter and therefore has not been exposed to the discounted payback, DPB, concept? Will she arrive at the same answer? For this individual, she either solves the problem using the Present Value Annuity (PVA) formula or uses the =NPER(…) formula in Excel. The PVA formula is:

$PV = A * \left(\dfrac{1 - \frac{1}{(1+i)^N}}{i} \right)$ where PV is the prevent value of the annuity, A = annuity, i = annual interest rate, and N = number of years.

$\left|-100\right| = 40 * [(1 - 1/(1.10)^N)/.10] + 10/(1.10)^N$, not forgetting the $10 salvage value recouped at the project's expiry.

Rearranging the terms, and isolating the N, we get:

$100 = 400(1 - 1/(1.10)^N) + 10/(1.10)^N$

$100 = 400 - 390/(1.10^N)$

$1.10^N = 390/300$

Now, take natural logarithm on both sides, we get:

$N*\ln 1.10 = \ln 1.30$. Solving for $N = \ln 1.30/\ln 1.10 = 2.75274126$ years. Let's call this number the time to breakeven, TTB.

Exactly the same result can be obtained by entering I/Y=10, PV= −100, PMT=40, FV=10, and press CPT N in the Texas Instrument® BA-II Plus Professional financial calculator. In Microsoft® Excel, we simply enter the following in a cell:

=NPER(10%, -100, 40, 10, 0) pressing the Enter key to get the exact same answer.

In a 365-day year, the difference between discount payback DPB and the time to breakeven TTB amounts to 96 days, i.e., $3.0154 - 2.752714 = .2627$ year ≈ 96 days.

What causes the difference? Perhaps, the difference was too small to arouse any interest or suspicion. To accentuate the difference, let's use another numerical example. In this example, let's choose the numbers such that we can conveniently apply the *Rule of 72*. Here are the assumed cash flows:

$CF_0 = -100$; $CF_1 = CF_2 = CF_3 = CF_4 = 0$, $CF_5 = 200$ and with interest remains at 10% p.a.

If we use the BA-II Plus Professional CF key, it will output 4.805255 years as the DPB. Again, Microsoft® Excel can be used to create the present values of the corresponding cash flows, and interpolation applied will yield exactly the same answer. However, if we use the time-value-money formula, of which Rule of 72 provides a quick estimate of, we will get:

$N = 72/10 = 7.2$ years via Rule of 72. If we choose to use the formula, we will get $200 = 100*(1.10)^N$ of which we can solve for N as $N = \ln 2/\ln 1.10 = 7.2725$ years.

Here, the difference between the DPB and the TTB of 2.467 years (=7.2725 − 4.805255) is now substantial and significant.

Axiom: Discounted payback, DPB $\neq$ TTB, time to breakeven. What is the difference between DPB and TTB? Given the cost of capital, the DPB must lie in the domain (0, N] for acceptable projects or else the project faces rejection. TTB, however, lies in the domains $[-\infty, 0)$ and $(0, \infty]$.

Illustrate the negative domain of TTB

Given: i=10% p.a., $CF_0 = -100$, $CF_1 = CF_2 = CF_3 = CF_4 = 0$, $CF_5 = 90$. To find the TTB, we solve algebraically: $90 = 100 (1.10)^N$ of which we find $N = \ln(.9)/\ln(1.10) = -1.105448714$. This negative N means at 10% annual interest rate, an investor should have received a loan of $90.00 at 1.105448714 years <u>before</u> she would then pay back the loan with $100.00 at t=0. Numerically, $90*(1.10)^{1.105448714} = 100.00$. The DPB for this project is non-existent since it is a money-losing proposition even at 0%, let alone at 10%, since $90 in t=5 at 0% interest rate will never be able to pay off $100 at t=0.

Illustrate the exclusion of 0 in the domain for TTB

Given: i=10% p.a., $CF_0 = -100$, $CF_1 = CF_2 = CF_3 = CF_4 = 0$, $CF_5 = 100$. To find the TTB, we solve algebraically $100 = 100(1.10)^N$ of which we get $\ln(1) = N*\ln(1.10)$. Isolating N, we get $N = 0/.09531018 = 0$. However, basic economic understanding forces us to recognize that for $100 invested today in exchange for $100 to be paid back 5 years from today, the rate of return is 0%. So, substituting 0% into the formula will get us to $100 = 100(1+0)^N$. Simplifying, we get $\ln 1 = N* \ln 1$. So, we land with the infamous millennia-old $0 \div 0$

conundrum in mathematics. To overcome such conundrum, the only practical solution for us is to exclude N=0 as a feasible domain. In Excel, =NPER(0%,0,-100,100,0) will return #DIV/0! while in the Texas Instruments® BA-II Plus Professional financial calculator, we will get an error message, Error 1 to be specific, when we enter I/Y=0, PV= −100, FV = 100, and press CPT N. The DPB for this project is also non-existent since the present value of the $100 to be received at t=5 at 10% p.a. is only $62.09 which is clearly insufficient to offset the $100 outlay incurred today.

In short, discount payback, DPB's domain is (0, N] where N is the project's life. Graphically,

O————————————————————●
0   N
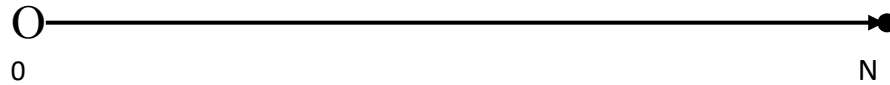
Time to breakeven, TTB's domains are [-∞,0) and (0,∞]. Graphically,

←————————————O————————————→
-∞   0   ∞
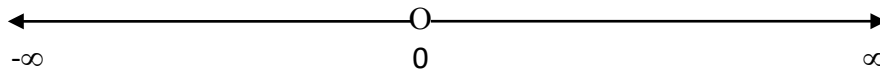
Significance: The above findings support the required teaching of capital budgeting to all business students regardless of their chosen academic discipline. The findings prove not only the insufficiency of time-value-money concept, but also the false sense of comfort the time-to-breakeven (which is an auxiliary of time-value-money concept) bestows in a simple project's cash flows. The discounted payback, which is the corollary of net present value, remains the better criterion of time measure in a project's evaluation. Any attempt to jettison the capital budgeting chapter with the excuse or hope to spend more time reinforcing on time-value-money chapter should bear in mind of this counter-example which illustrates the shortcoming the the time to breakeven.

Reference
Osborne, Michael J., 2010, A resolution of the NPV-IRR debate?, The Quarterly Review of Economics and Finance, vol. 50, pp. 234-9. DOI: 10.1016/j.qref.2010.01.002